PRINTED IN POLAND

Über eine hinreichende Bedingung für die Existenz einer invarianten Metrik in homogenen Räumen

von

A. GOETZ

Vorgelegt von K. KURATOWSKI am 22. Juni 1955

Nach einem Satze von Kakutani ([1] und [2], Seite 14) ist in jeder topologischen Gruppe mit dem ersten Abzählbarkeitsaxiom eine linksinvariante und eine rechtsinvariante Metrik vorhanden. Hier soll eine hinreichende Bedingung dafür angegeben werden, daß auch der durch eine abgeschlossene Untergruppe H von G bestimmte homogene Raum G/H invariant metrisierbar sei. Es wird vorausgesetzt, daß für G das erste Abzählbarkeitsaxiom gilt.

Sei die Entfernung ϱ in G linksinvariant, d. h. es bestehe die Identität

$$\varrho(x,y) = \varrho(zx,zy) \quad \text{für alle} \quad x,y,z \in G.$$

Wir setzen für die Mengen $A, B \subset G$ und das Element $x \in G$

$$\varrho(A,B) = \inf_{z_1 \in A, z_2 \in B} \varrho(z_1,z_2) \quad \text{und} \quad \varrho(x,A) = \inf_{z \in A} \varrho(x,z).$$

Dann gilt der

HILFSSATZ. *Aus jeder der Bedingungen*

(A) *H ist Normalteiler von G,*

oder

(B) $\varrho(xz,yz) = \varrho(x,y)$ *für beliebige* $x,y \in G$ *und jedes* $z \in H$,

folgt

(C) $\varrho(x_0 H, y_0 H) = \varrho(x_0, y_0 H)$ *für irgendwelche* $x_0, y_0 \in G$.

Beweis. 1°. Aus (A) folgt (C). Multiplizieren wir in

$$\varrho(x_0 H, y_0 H) = \inf_{z_1, z_2 \in H} \varrho(x_0 z_1, y_0 z_2)$$

die beiden Elemente $x_0 z_1, y_0 z_2$ links mit $x_0 z_1^{-1} x_0^{-1}$, so erhalten wir, zufolge der Linksinvarianz der Metrik ϱ,

$$\varrho(x_0 H, y_0 H) = \inf_{z_1, z_2 \in H} \varrho(x_0, x_0 z_1^{-1} x_0^{-1} y_0 z_2)$$

32*

und daraus weiter

$$\varrho(x_0 H, y_0 H) = \inf_{z_1, z_2 \in H} \varrho(x_0, y_0 y_0^{-1} x_0 z_1^{-1} x_0^{-1} y_0 z_2) = \inf_{z \in H} \varrho(x_0, y_0 z) = \varrho(x_0, y_0 H),$$

weil H nach Voraussetzung Normalteiler ist und deswegen das Element $z = y_0^{-1} x_0 z_1^{-1} x_0^{-1} y_0 z_2$ zusammen mit z_1 und z_2 das ganze H durchläuft.

2°. Aus (B) folgt (C). Nach (B) ist

$$\varrho(x_0 H, y_0 H) = \inf_{z_1, z_2 \in H} \varrho(x_0 z_1, y_0 z_2) = \inf_{z_1, z_2 \in H} \varrho(x_0, y_0 z_2 z_1^{-1}) =$$

$$= \inf_{z \in H} \varrho(x_0, y_0 z) = \varrho(x_0, y_0 H).$$

SATZ. *Ist eine der Bedingungen* (A) *oder* (B) *erfüllt, so existiert im homogenen Raume G/H eine invariante Metrik d, d. h. eine solche Metrik, daß*

$$d(uX, uY) = d(X, Y) \quad \textit{für beliebige} \quad X, Y \in G/H \textit{ und } u \in G.$$

Beweis. Es bedeute φ die natürliche Abbildung von G auf G/H und es seien $X = \varphi(x)$ und $Y = \varphi(y)$ die durch die Elemente x und y von G erzeugten Punkte des Raumes G/H. Setzen wir

$$d(X, Y) = \varrho(xH, yH).$$

Es ist evident, daß diese Definition von der Wahl der erzeugenden Elemente x und y nicht abhängt und daß aus der Linksinvarianz der Metrik ϱ die Invarianz von d folgt. Offenbar ist auch

$$d(X, X) = 0, \quad d(X, Y) > 0, \quad \text{wenn} \quad X \neq Y, \quad d(X, Y) = d(Y, X).$$

Es bleibt nur noch die Dreiecksungleichung

$$d(X, Y) + d(Y, Z) \leqslant d(X, Z)$$

festzustellen, um bewiesen zu haben, daß d eine invariante Metrik in G/H ist.

Nach dem Hilfssatz haben wir

$$d(X, Y) = \inf_{u_1 \in H} \varrho(y, xu_1), \quad d(Y, Z) = \inf_{u_2 \in H} \varrho(y, zu_2),$$

wo x, y, z erzeugende Elemente der Punkte X, Y, Z sind. Daher haben wir weiter

$$d(X, Y) + d(Y, Z) = \inf_{u_1, u_2 \in H} [\varrho(y, xu_1) + \varrho(y, zu_2)] \leqslant$$

$$\leqslant \inf_{u_1, u_2 \in H} \varrho(xu_1, zu_2) = d(X, Z),$$

w.z.b.w. Da die durch die Metrik d erzeugte Topologie in G/H mit der natürlichen zusammenfällt, ist damit der Satz bewiesen.

Die Bedingung (B) ist immer erfüllt, wenn H bikompakt ist.

Man bemerke, daß der erste Teil dieses Satzes unmittelbar aus dem Kakutanischen Satze folgt, da für einen Normalteiler H der Raum G/H eine topologische Gruppe mit dem ersten Abzählbarkeitsaxiom ist.

Aus unserem Satz ergibt sich sofort das

KOROLLAR. *Sei* H_0 *ein in* H *enthaltener Normalteiler von* G. *Ist für die Gruppe* G/H_0 *und ihre Untergruppe* H/H_0 *die Bedingung* (B) *erfüllt, so gibt es in* G/H *eine invariante Metrik.*

Der Frage der notwendigen Bedingungen wird eine andere Arbeit des Verfassers gewidmet.

MATHEMATISCHES INSTITUT DER POLNISCHEN AKADEMIE DER WISSENSCHAFTEN.

SCHRIFTTUM

[1] S. Kakutani, *Über die Metrisation der topologischen Gruppen,* Proc. Imp. Acad. Japan, **12** (1936), 82.

[2] M. I. Graieff, *Teoria topologitscheskich grupp* I, Usp. Mat. Nauk, **5**, 2 (1950), 3.

Über gemischte Rand- und Anfangswertprobleme im Grossen für eine Klasse von Gleichungssystemen auf differenzierbaren Mannigfaltigkeiten. Eine Begründung der Fouriermethode

von

K. MAURIN

Vorgelegt von K. KURATOWSKI am 27. Juni 1955

In dieser Note wird gezeigt, wie mit Hilfe des Operatorenkalküls von Stone -, von Neumann eine grosse Anzahl gemischter Rand- und Anfangswertprobleme, sowie das Cauchy-Problem für eine weite Klasse formalselbstadjungierter Gleichungssysteme auf differenzierbaren Mannigfaltigkeiten, in äusserst einfacher Weise gelöst werden kann. Es wird auch bewiesen, dass die verallgemeinerte Lösung des Problems von den Anfangsbedingungen in stetiger Weise abhärgt. Gestützt auf die Ergebnisse der von Lars Gårding vor Kurzem entdeckten Theorie der Eigenfunktionsentwicklungen wird zugleich eine Begründung der verallgemeinerten Fouriermethode gegeben, die die Methode des Fourierintegrals und der Fourierreihen umfasst.

Die hier wiedergegebenen Ergebnisse enthalten als Sonderfälle diejenigen von K. Yosida [3] und Ladyschenskaja [4]. Die Idee, das Operatorenkalkül der Funktionen eines selbstadjungierten Operators anzuwenden, stammt von Yosida (im Falle des Cauchy-Problems für die Wellengleichung im Riemann'schen Raume). Etwas später wurde sie von F. E. Browder für ein parabolisches System angewandt [1].

Die Methode von Browder ist jedoch schon im Falle der Wellengleichung im euklidischen Raume nicht anwendbar. Unsere Methode bildet eine Weiterführung und Vereinfachung derjenigen von K. Yosida und gestattet u.a. auch einen einfachen Beweis des Satzes von Browder.

1. Definitionen. Ω_n ist eine offene differenzierbare n-dimensionale Mannigfaltigkeit mit oder ohne Rand (im ersten Falle wird man die Lösung der gemischten Rand- und Anfangswertprobleme und im zweiten — nur die des Cauchy-Problems erhalten).

$x = (x_1, \ldots, x_n)$ bedeuten die lokalen Koordinaten der Ω_n.

$A = A_x = A_x^+$ ist ein formalselbstadjungiertes elliptisches System der Ordnung $\sigma(\sigma \geqslant 1)$ auf Ω_n mit hinreichend regulären Koeffizienten bzgl. der Definitionen des adjungierten Systems A^+ and der Voraussetzungen über die Koeffizienten von A (vgl. [5]).

A_0 bedeutet die Einschränkung des Operators A auf $C_c^{\infty,r}(\Omega_n)$ (beliebig oft in Ω_n differenzierbare Vektorfelder mit r-Komponenten mit kompakten Trägern in Ω_n).

$L^{2,r}(\Omega_n)$ sei der Hilbert'sche Raum der messbaren, quadratisch in Ω_n integrierbaren Vektorfelder mit dem Skalarprodukt

$$(u, v) = \sum_{i=1}^{r} \int_{\Omega_n} u_i(x) \overline{v_i(x)} \, dx.$$

$A_1 \supset A_0$ ist eine selbstadjungierte Fortsetzung von A_0.

$D(A_1)$ bezeichnet den Definitionsbereich von A_1.

Es gilt: $A_0^* \supset A_1$. Das Element u von $L^{2,r}(\Omega_n)$ heisst die schwache Lösung von $Au = h$, wenn $A_0^* u = h$ (vgl. [5]).

Von A_1 wird vorausgesetzt, dass

(1.1) $$(-A_1 \varphi, \varphi) \geqslant a(\varphi, \varphi),$$

wobei $a > -\infty$ (Halbbeschränktheit des Spektrums von A_1!).

2. Formulierung und Lösung der gemischten Probleme und des Cauchy-Problems.

Gesucht ist eine Lösung $u(x, t)$ des Systems

(2.1) $$\frac{\partial^2 u(x, t)}{\partial t^2} = A_x u(x, t),$$

die für $t = 0$ die Anfangsbedingungen

2.2) $$u(x, 0) = f(x); \qquad \frac{\partial u(x, 0)}{\partial t} = g(x)$$

und eine Randbedingung

(2.3) $$u(-, t) \, \epsilon \, D(A_1)$$

erfüllt.

Bemerkung: im Falle einer offenen Mannigfaltigkeit Ω_n ohne Rand, hat man mit dem Cauchy-Problem zu tun.

Nach K. Yosida [3] wird als operatortheoretische Variante von (2.1)—(2.3) die folgende Aufgabe angeführt:

Gesucht is eine Vektorfunktion $u(-, t) \subset D(A_1)$, die der Gleichung

(2.1') $$\frac{d^2 u(-, t)}{dt^2} = A_1 u(-, t) \qquad \left(\frac{d}{dt} \text{ bedeutet die starke Ableitung} \right)$$

für $t \geq 0$ genügt und ausserdem die Anfangsbedingungen

$$(2.2') \qquad u(-,0) = f \in D(A_1), \qquad \frac{du}{dt}(-,0) = g \in D(A_1^{1/2})$$

erfüllt. $u(-,t)$ wird auch die verallgemeinerte (operatortheoretische) Lösung des Problems genannt.

Es gilt der folgende

HAUPTSATZ.

a) *Das Problem* (2.1'), (2.2') *besitzt eine (einzige) Lösung*

$$(2.4) \qquad u(-,t) = \int\limits_{a > -\infty}^{\infty} \cos \lambda^{1/2} t \, d \, E(\lambda) f + \int\limits_a^{\infty} \lambda^{-1/2} \sin \lambda^{1/2} t \, d E(\lambda) g,$$

wobei $-A_1 = \int\limits_{a > -\infty}^{\infty} \lambda \, dE(\lambda)$ *die Spektraldarstellung von* $-A_1$ *ist.*

b) *Wenn* $f \in D(A_1^m), g \in D(A_1^{m-1/2})$, *wo* $m > \dfrac{n}{2\sigma} + 1$, *dann ist* $u(-,t) \in C_-^{\sigma,r}(\Omega_n)$.

c) *Wenn* $f \in D(A_1^{m+1})$, $g \in D(A_1^m)$, $m > \dfrac{n}{2\sigma} + 1$, *dann ist* $u(-,t)$ *in* $\Omega_n \times T$ *äquivalent einem Vektorfeld* $\tilde{u}(x,t)$ (*wobei* $T = \{t : t \geq 0\}$) *mit den Eigenschaften:* $\tilde{u}(-,t) \in C^{\sigma,r}(\Omega_n)$, $\tilde{u}(x,-) \in C^{2,r}(T)$, *d.h.* $\tilde{u}(x,t)$ *ist die (klassische) Lösung des Problems* (2.1)—(2.3).

d) *Wenn* $f, g \in D(A_1^q)$ *für jedes* $q = 1, 2, \ldots$, *dann ist* $u(-,t)$ *äquivalent dem Vektorfeld* $\tilde{u}(x,t) \in C^{\infty,r}(\Omega_n \times T)$.

Diese letzte Aussage enthält als Spezialfall den Satz von K. Yosida [5]. Im Falle d) wird vorausgesetzt, dass die Koeffizienten von A_x beliebig oft differenzierbar sind.

Beweis.

ad a)

$$\frac{du}{dt}(-,t) = -\int\limits_a^{\infty} \lambda^{1/2} \sin \, dE(\lambda) f + \int\limits_a^{\infty} \cos \lambda^{1/2} t \, dE(\lambda) g,$$

$$-\frac{d^2 u}{dt^2}(-,t) = \int\limits_a^{\infty} \lambda \cos \lambda^{1/2} dE(\lambda) f + \int\limits_a^{\infty} \lambda^{1/2} \sin \lambda^{1/2} t \, dE(\lambda) g = -A_1 u(-,t),$$

woraus die Behauptung folgt.

ad b) Wegen der über f, g gemachten Voraussetzungen ist

$$u(-,t) \in D(A_1^m) \subset D(A_0^{*m}) \qquad \text{für} \qquad m > \frac{n}{2\sigma} + 1,$$

woraus sich die Behauptung aus dem in [5] bewiesenen Hauptsatz ergibt.

ad c) und d) Wenn wir $A_1^m u(-,t) = h(-,t)$ setzen, folgt aus den gemachten Voraussetzungen, dass $h(-,t)$ und $u(-,t)$ zweimal stark differenzierbar sind; da $A_0^{*m} u(-,t) = h(-,t)$, folgt ebenfalls aus dem oben herangezogenen Hauptsatz, dass es für jedes $\mathring{x}$ eine solche Koordinatenumgebung $V(\mathring{x}) \ni \mathring{x}$ gibt, dass für jedes $x \in V(\mathring{x})$, $t \geqslant 0$:

$$(2.5) \qquad u_i(x;t) = \int_{V(\mathring{x})} \sum_{\nu=1}^r u_r(z,t)\, r_{ri}(z,x)\, dz + \int_{V(\mathring{x})} \sum_{\nu=1}^r h_r(z,t)\, \omega_{ri}(z,x)\, dz$$

(fast überall in $V(\mathring{x})$), wobei $r_{ik}(z,x)$, $\omega_{ik}(z,x)$ $i,k = 1,\ldots,r$ für $z \neq x$ regulär in $V(\mathring{x}) \times V(\mathring{x})$ sind. Da $r_{ik}(x,-)$, $\omega_{ik}(x,-) \in L^2(V(\mathring{x}))$, ist die rechte Seite von (2.5) zweimal nach t stetig differenzierbar: dass sie σ-mal nach x stetig differenzierbar ist, wurde in [5] bewiesen.

Die unendlich oft wiederholte Anwendung derselben Schlussweise erledigt auch den Fall d). Der Beweis des Hauptsatzes ist also vollständig erbracht.

3. Der Fall des „parabolischen" Systems:

$$(3.1) \qquad \frac{\partial v(x,t)}{\partial t} = A_x v(x,t), \qquad (3.2) \qquad v(-,t) \underset{t \to 0}{\to} g \in L^{2,r}(\Omega_n)$$

führt mit dem Ansatz

$$(3.3) \qquad v(-,t) = \int_{a > -\infty}^{\infty} e^{-\lambda t}\, dE(\lambda)\, g, \qquad t > 0$$

zu dem

SATZ VON BROWDER [1]. *Es existiert eine Lösung* $v(x,t)$ *der Gleichung* (3.1), *wobei* $v(x,-) \in C^{\infty,r}(R)$ *für* $x \in \Omega_n$, $R = \{t : t > 0\}$; *für* $t > 0$ $v(-,t) \in D(A_1)$ *(das ist eine Randbedingung, vgl. die Bemerkung in 2.),* $v(-,t) \to g$ *in* $L^{2,r}(\Omega_n)$, *wenn* $t \to 0$ *(das ist eine Anfangsbedingung).*

Beweis wie oben: die Anwendbarkeit der Schlussweise aus d) ist gesichert durch die schnelle Abnahme von $e^{-\lambda t}$ für $\lambda \to \infty$, $t > 0$, w.z.b.w.

4. Die Korrektheit des Problems.

Die operatortheoretische Variante des Problems von (2.1)—(2.3) *ist sachgemäss in dem Sinne, dass:*

　　1^0 *sie eine und nur eine Lösung besitzt,*

　　2^0 *die Lösung von den Anfangswerten stetig abhängt.*

Einen überaus einfachen Beweis von 1^0 — den wir hier nicht anführen — verdanke ich der Mitteilung von Prof. Dr. Ryll-Nardzewski.

Beweis von 2^0. Es sei $\overset{i}{u}(-,t)$ die Lösung von (2.1'), (2.2') bei den Anfangswerten $\overset{i}{f}, \overset{i}{g}$ $(i=1,2)$; dann folgt aus (2.4):

$$\|\overset{1}{u}(-,t)-\overset{2}{u}(-,t)\|^2 \leqslant \int\limits_a^\infty |\cos \lambda^{1/2}t|^2 d\| E(\lambda)(\overset{1}{f}-\overset{2}{f})\|^2 +$$

$$+ \int\limits_a^\infty |\lambda^{-1/2}\sin \lambda^{1/2}t|^2 d\| E(\lambda)(\overset{1}{g}-\overset{2}{g})\|^2 < C\{\|\overset{1}{f}-\overset{2}{f}\|^2 + \|\overset{1}{g}-\overset{2}{g}\|^2\},$$

wo C eine positive (von t unabhängige) Konstante ist, w.z.b.w.

5. Eine Begründung der Fouriermethode.

Wie L. Gårding gezeigt hat [2], gilt für die Spektralschar $E(\lambda)$ des Operators $-A_1 = \int\limits_a^\infty \lambda dE(\lambda)$ die folgende Darstellung mittels der Eigenfunktionen des Operators A_x:

(5.1) $$[E(\lambda_1)-E(\lambda_2)]f(y) = \int\limits_{\lambda_1}^{\lambda_2} \sum\limits_{j=1}^{N(\lambda)} \theta^j(y,\lambda)\hat{f}^j(\lambda)\,d\tau(\lambda),$$

wobei $\hat{f}^j(\lambda) = \text{l.i.m.} \sum\limits_{i=1}^r \int\limits_{\Omega_n} \theta^j_i(x,\lambda)f_i(x)\,dx$, wo $\theta^j(-,\lambda) \epsilon\, C^{\sigma,r}(\Omega_n)$ und $A_x\theta^j(x,\lambda) = \lambda\theta^j(x,\lambda)$ $j=1,2,\ldots,N(\lambda)$. $N(\lambda)$ kann auch unendlich sein. $\theta^j(x,\lambda)$ sind also Eigenfunktionen des Operators A_x. $\tau(\lambda)$ ist ein nichtnegatives Mass auf der reellen Achse (a, ∞).

Wenn wir (5.1) und das Entsprechende für $[E(\lambda_1)-E(\lambda_2)]g(y)$ in (2.4) bzw. (3.3) einsetzen, erhalten wir die Losung der Probleme in der Gestalt der verallgemeinerten Fourier-Integrale (die sich im Falle des reinen Punktspektrums auf die Fourierreihen reduzieren). Wir haben also auf diese Weise eine Begründung der Fouriermethode gewonnen, die für jedes beliebige Spektrum anwendbar ist.

MATHEMATISCHES INSTITUT DER POLNISCHEN AKADEMIE DER WISSENSCHAFTEN.

SCHRIFTTUM

[1] F. E. Browder, *Eigenfunktion expansions for singular elliptic operators.* II. *The Hilbert space argument; parabolic equations on open manifolds.* Proc. Nat. Acad. U.S.A. **40** (1954), 459.

[2] L. Gårding, *Eigenfunction expansions connected with elliptic differential operators,* Skand. Mat.-Kongr., Lund, **12** (1953), 44.

[3] K. Yosida, *On Cauchy's problem in the large for wave equations,* Proc. Japan Acad., **28** (1952), 396.

[4] O. A. Ladyschenskaja, *Smjeschannaja sadatscha dla giperbolitscheskovo urawnjenia,* Moskau 1953.

[5] K. Maurin, *Der Fundementalsatz über schwache Lösungen der allgemeinen linearen Systeme der elliptischen Differentialgleichungen beliebiger Ordnung,* Bull. Acad. Polon. Sci., Cl. III, **2** (1954), 457.

On the Bases of Modules over a Principal Ideal Ring

by

A. W. MOSTOWSKI and E. SĄSIADA

Presented by K. KURATOWSKI on July 7, 1955

1. The purpose of this note is to prove that a module G over a principal ideal ring R has a basis, if for some $a \in R$ the submodule aG composed of all elements $ag (g \in G)$ has a basis. Using this result, we give a simple proof of the well known basic theorem for a finitely generated module over a principal ideal ring and of one of Prüfer's theorems.

In the case of a ring of integers this result was at first obtained by L. Fuchs in the paper [1] in consequence of his "basic criterion" which, however, has no direct generalisation for modules.

2. In this section we summarise the known facts which we need:
(1) In a principal ideal ring R every element (neither 0 nor a unit) is expressible as a product of prime elements and the expression is unique, up to order and insertion of units.
(2) If a is a prime element, then the ring $R/(a)$ is a field. *)
(3) Any finitely generated torsion-free module over a principal ideal ring has a basis.

Let R be a principal ideal ring and G an R-module. We have then:
(4) If the order of every $g \in G$ is a prime ideal (a), then the module G has a basis and every submodule of G is a direct summand of G.
(5) Any torsion module over a principal ideal ring is a direct sum of primary submodules. **)

3. THEOREM. *Let R be a principal ideal ring and G an R-module. If for some $a \in R$ the submodule aG has a basis, then likewise G has a basis.*

Proof. Owing to (1) we can restrict our proof to the case in which a is a prime of R. Let a be a prime of R such that aG has a basis. By (5) there exists a basis B of aG such that each $b \in B$ has the order (0) or (β^n), where β is a prime of R, and n a positive integer.

---

*) By (a) we denote the principal ideal of the element a.
**) For the theorems (3), (4) and (5) we refer to [2].

For every element $\boldsymbol{b}$ of $\boldsymbol{B}$ we choose an element $\boldsymbol{a}$ of $\boldsymbol{G}$ with $a\boldsymbol{a}=\boldsymbol{b}$ in such a way that, if $\boldsymbol{b}$ is of order (β^n) and $\beta\neq a$, then $\boldsymbol{a}$ belongs to the cyclic group of $\boldsymbol{b}$. Such a choice is possible by (2).

It is easy to see that the set $\boldsymbol{A}$ of elements $\boldsymbol{a}$ thus chosen forms an independent set of module $\boldsymbol{G}$. Let $\boldsymbol{G}_1$ be the submodule of $\boldsymbol{G}$ generated by elements of $\boldsymbol{A}$. $\boldsymbol{A}$ is a basis for $\boldsymbol{G}_1$.

The set $\boldsymbol{G}^*$ of all elements $\boldsymbol{g}\in\boldsymbol{G}$ such that $a\boldsymbol{g}=0$ forms a submodule. By (4) the submodule $\boldsymbol{G}_1\cap\boldsymbol{G}^*$ is a direct summand of $\boldsymbol{G}^*$, i. e.

$$(*) \qquad\qquad \boldsymbol{G}^*=(\boldsymbol{G}_1\cap\boldsymbol{G}^*)+\boldsymbol{G}_0,$$

and the submodule $\boldsymbol{G}_0$ has a basis.

To complete the proof of our theorem, it is sufficient to show that $\boldsymbol{G}=\boldsymbol{G}_0+\boldsymbol{G}_1$. Clearly, $\boldsymbol{G}_0\cap\boldsymbol{G}_1=\{0\}$. Let $\boldsymbol{g}\in\boldsymbol{G}$; then $a\boldsymbol{g}\in a\boldsymbol{G}$; hence

$$a\boldsymbol{g}=\gamma_1\boldsymbol{b}_1+\ldots+\gamma_n\boldsymbol{b}_n=a(\gamma_1\boldsymbol{a}_1+\ldots+\gamma_n\boldsymbol{a}_n)$$

and

$$a(\boldsymbol{g}-\gamma_1\boldsymbol{a}_1+\ldots+\gamma_n\boldsymbol{a}_n)=0.$$

Therefore we have

$$\boldsymbol{g}-\gamma_1\boldsymbol{a}_1+\ldots+\gamma_n\boldsymbol{a}_n=\boldsymbol{g}^*\in\boldsymbol{G}^*.$$

Following (*) $\boldsymbol{g}^*\in\boldsymbol{G}_0+\boldsymbol{G}_1$, and as $\gamma_1\boldsymbol{a}_1+\ldots+\gamma_n\boldsymbol{a}_n\in\boldsymbol{G}_1$, we have finally $\boldsymbol{g}\in\boldsymbol{G}_0+\boldsymbol{G}_1$.

4. In this section we give, as application of the above proved theorem, simple and short proofs of two known theorems.

COROLLARY 1. *Any finitely generated module* $\boldsymbol{G}$ *over a principal ideal ring* $\boldsymbol{R}$ *has a basis.*

Proof. There exists an element $a\in\boldsymbol{R}$ such that every non-zero element in $a\boldsymbol{G}$ has the order (o), and so this corollary follows from (3) and from our theorem.

COROLLARY 2. (Prüfer). *Let* $\boldsymbol{R}$ *be a principal ideal ring and* $\boldsymbol{G}$ *an* $\boldsymbol{R}$-*module. If* $a\boldsymbol{G}=\{0\}$ *for some* $a\in\boldsymbol{R}$, *then* $\boldsymbol{G}$ *has a basis.*

Proof. Since the zero module has a trivial basis, this corollary is an obvious consequence of our theorem.

For helpful suggestions in writing this note we wish to thank Professor J. Łoś.

INSTITUTE OF MATHEMATICS, POLISH ACADEMY OF SCIENCES.

REFERENCES

[1] L. Fuchs, *The direct sum of cyclic groups*, Acta Math. Hung. **3** (1952), 177.
[2] I. Kaplansky, *Infinite Abelian Groups*, University of Michigan Press, Ann Arbor, 1954.

Über eine Definition des Wertes einer Distribution

von

S. ŁOJASIEWICZ, J. WLOKA und Z. ZIELEŹNY

Vorgelegt von S. MAZUR am 25. Juli 1955

In dieser Note führen wir eine Definition des Wertes einer Distribution in einem Punkte ein, die uns in einfacher Weise bestimmte Integrale einer Distribution, insbesonders auch die Fourierschen Koeffizienten einer periodischen Distribution zu definieren erlaubt. Unsere Begründung und Herleitung der wichtigsten Eigenschaften der periodischen Distributionen ist sehr naheliegend und benutzt keine Hilfsmittel der Funktionalanalysis. *)

Wir wollen Distributionen rein symbolisch mit $f(x)$, also so wie Funktionen, bezeichnen. Bei dieser Bezeichnungsweise ist der Sinn des Symbols $f(ax+b)$, für $a \neq 0$ klar.

Satz I. *Wenn der Grenzwert* $\lim\limits_{\varepsilon \to 0} f(\varepsilon x + x_0)$ *existiert, so ist er eine konstante Funktion.*

Der Wert dieser konstanten Funktion wird als der Wert der Distribution im Punkte x_0 bezeichnet und durch das Symbol $f(x_0)$ gekennzeichnet. (Diese Definition stammt von S. Łojasiewicz). Falls die Distribution $f(x)$ in einer Umgebung von x_0 eine in x_0 stetige Funktion ist, dann stimmt dieser Wert mit dem üblichen Wert überein.

Das bestimmte Integral

$$\int_a^b f(x) \cdot dx$$

definieren wir als den Wert der Distribution $F(x+b) - F(x+a)$ im Punkte 0, wobei $F'(x) = f(x)$. Dieses Integral ist nur dann definiert, wenn dieser Wert existiert, also insbesonders wenn die Werte $F(b)$ und $F(a)$ existieren;

---

*) Die Beweise der in dieser Note angegebenen Sätze werden in einer ausführlichen Arbeit in den Rozprawy Matematyczne erscheinen.

wir haben dann

$$\int_a^b f(x) \cdot dx = F(b) - F(a).$$

Es ist klar, dass ein so definiertes Integral eine Zahl ist, die von der Wahl der "primitiven" Distribution $F(x)$ nicht abhängt.

Satz II. *$F(x)$ sei irgendein unbestimmtes Integral der Distribution $f(x)$. Hat die Distribution $f(x)$ im Punkt x_0 einen Wert, so hat auch die Distribution $F(x)$ im Punkt x_0 einen Wert.*

Aus Satz II folgt:

Satz III. *Wenn die Distribution $f(x)$ in den Punkten a und b einen Wert annimmt, so existiert das bestimmte Integral $\int_a^b f(x) \cdot dx$.*

Wir nennen eine Distribution $f(x)$ periodisch mit der Periode 2π, wenn $f(x+2\pi) = f(x)$ ist.

Satz IV. *Das Integral $\int_0^{2\pi} f(x) \cdot dx$ existiert für jede periodische Distribution $f(x)$.*

Ist $f(x)$ periodisch, so sind auch die Distributionen $f(x)\cos nx$, $f(x)\sin nx$ und $f(x)e^{-inx}$ periodisch.

Es existieren also die Integrale:

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\,dx, \qquad b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx\,dx$$

(*) und

$$c_n = \frac{1}{2\pi} \int_0^{2\pi} f(x) e^{-inx} \cdot dx.$$

Es ist zu betonen, dass mit den obigen Bezeichnungen der Fouriersche Entwicklungsatz in üblicher Weise formuliert werden kann:

Satz V. *Jede periodische Distribution lässt sich in eine konvergente Reihe*

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

oder

$$\sum_{n=-\infty}^{\infty} c_n e^{inx}$$

entwickeln, wobei die Koeffizienten $a_n, b_n,$ und c_n durch die Formeln (*) definiert sind.

Die Eindeutigkeit dieser Entwicklungen lässt sich genau so, wie im Falle der gleichmässigen Konvergenz, beweisen.

Aus den obigen Betrachtungen zeigt sich, dass die hier auf eine Anregung von Prof. J. Mikusiński hin eingeführte Schreibweise sehr bequem ist, weil alle Rechnungen mit Distributionen vollständig den üblichen Rechnungen mit Funktionen analog sind und nur eine neue Interpretation brauchen.

MATHEMATISCHES INSTITUT DER POLNISCHEN AKADEMIE DER WISSENSCHAFTEN.

SCHRIFTTUM

[1] L. Schwartz, *Théorie des distributions* II, Paris, 1951.
[2] R. Sikorski, *A definition of the notion of distribution*, Bull. Acad. Polon. Sci., Cl. III, **2** (1954), 209.

A Statement Concerning Researches on the Motion of the P/Comet 1906 IV (Kopff)

by

F. KĘPIŃSKI

Presented by W. DZIEWULSKI on June 29, 1955

The author presents a brief account of the actual state of his researches concerning the motion of the P/Comet 1906 IV (Kopff) and a working schedule for the next few years.

The Comet was discovered by Professor Kopff in 1906. It was unfortunately not observed again when it next returned in 1913, but in 1919 its favourable position enabled it to be observed by many.

The author began his researches on this Comet in 1926 [1], [2]. In his paper [3] the author connected the 1919 reappearance of this Comet, about which there were numerous observations, with that of 1926 which was picked up only a few times. Thus the first exact system of elements found in this way was used as a basis for the author's further computations. His next paper, published in 1936 [4], contained the more exact linking of three reappearances of this Comet in 1919, 1926 and 1932/33, with mean quadratic error $\pm 3.0''$ of one normal place with the weight 1. It is worthy of note that the perturbations of all great planets from Venus to Saturn were also taken into account. In this way it was possible to consider the deduced S_3 system of elements as quite accurate for the precise computations of perturbations in the motion of the Comet during the subsequent period, 1932−1939 [5]. Confirmation of the great exactness of the basic S_3 system was obtained by comparing with the observations the author's accurate ephemeris of the Comet for its reappearance in 1939, since the differences, $\cos \delta \cdot \varDelta a$ and $\varDelta \delta$, barely amounted to 0.5^s and $0.1'$ respectively [8].

Thus, the author was justified in using henceforth the same S_3 system for his further computations of the motion of the Comet for 1939−1945, particularly since our country, culturally isolated at that time, was cut off from the requisite scientific sources.

As mentioned in one of the publications quoted, all the author's papers containing the elaboration of the theory of this Comet, except the exact ephemeris for 1945 [6], perished by fire. Fortunately, a few

copies had been reproduced and distributed outside Warsaw. Thus it was saved, to reach him as he was performing further computations on the application of Bobone's elements. The explanation of these various circumstances, responsible for considerable deviations from observations in his initial ephemeris 1951/52 [7], is, together with the necessary corrections, given in the recent publications [9], [10].

It has already been mentioned above that all the computations during the period 1919—1926—1932/33 were made with the S_3 system of elements, which was also taken as a basis for the computation in the next period, 1932—1945. As regards the perturbations for the period 1945—1952 [7], [10], they need certain corrections which will be effected by taking into account: (1) the new system of elements deduced in integral compatibility with the ephemeris for 1945, which was luckily saved from destruction, as stated above; and, (2) differential corrections of perturbations for the period 1945—1952 by the replacement of Bobone's elements by the author's.

The high degree of agreement as between theory and observation in 1939, a very satisfactory degree for 1945 [8], and lastly, certain discrepancies in 1951 (as a provisional result, taking into account only point (1) $0''\!.1$ and $0''\!.3$ were obtained) justify regarding as certain that the S_3 system will not undergo any marked alterations enabling the last apparition to be linked with the preceding ones.

It should be noted that the orbit of the Comet underwent in 1942 considerable variation due to the approach to Jupiter amounting to 0.57 a.u. Consequently, even the small variations of the S_3 system might produce measurable differences O-C after 1939.

Since the Comet will pass its perihelion in 1957/58, the corresponding ephemeris should be computed in advance. The Comet — it should be remembered — again approached close to Jupiter in the first quarter of 1954. The relative computations are now in progress by M. Bielicki (Warsaw University) and the author. The former is at present computing by the method of variation of arbitrary constants, the perturbations in the motion of the Comet for the period 1952—1958 due primarily to Jupiter and Saturn. Thereafter those, due to Venus, the Earth, Mars and Uranus will be taken into consideration. The perturbations due to Jupiter increased rapidly as the Comet was entering into the sphere of Jupiter's activity. The author of the present note is continuing his researches over the connection of the reappearance of the Comet in 1951 with the preceding appearances, in the first instance those of 1939, 1945 and 1951.

The preliminary calculations of M. Bielicki have confirmed the author's tentative prevision of the close approach of the Comet to Jupiter of the order 0.18 a.u. The very considerable variations in the position of its node line are to be expected.

DEPARTMENT OF PRACTICAL ASTRONOMY, INSTITUTE OF TECHNOLOGY, WARSAW

REFERENCES

[1] F. Kępiński, *Retour en 1926 de la Comète Kopff au Soleil*, Circ. of the Astr. Obs. of Warsaw Univ., 1926.

[2] — *La Comète Kopff en 1919*, Publ. of the Astr. Obs. of Warsaw Univ.. 2 (1926).

[3] — *Die Bewegung des periodischen Kometen Kopff (1906 e)*, I Teil, Publ. de l'Inst. d'Astr. Pr. de l'Ec. Pol. de Vars., 1932, No. 8.

[4] — *The motion of the Periodic Comet Kopff (1936 e)*, p. II, ibid.,1936, No. 16.

[5] — *The motion of the Periodic Comet Kopff (1906 e)*, p. III, ibid., 1938, No. 20.

[6] — *The motion of the Periodic Comet 1906 IV*, p. IV, ibid., 1946 No. 23.

[7] — *The motion of the Periodic Comet 1906 IV*, p. V, ibid., 151, No. 26.

[8] — *La Comète Périodique 1906 IV (Kopff) à son apparition en 1939 et 1945*, p. VI, ibid., 1951, No 26.

[9] — *O pewnym zagadnieniu z teorii poprawiania orbit eliptycznych*, Sprawozd. P.T.A., 1951, zesz. 3.

[10] — *Apparition de la Comète Périodique 1906 IV (Kopff) en 1951*, Acta Astronomica 1955.

The Action of T 40 on Drug Resistance of Acid-Fast Bacilli *in vitro*

by

M. BURACZEWSKA and W. MANOWSKA

Presented by T. URBAŃSKI on May 16, 1955

Our aim in undertaking this work was to study the formation of bacterial drug resistance *in vitro* under the influence of streptomycin (SM), INH, PAS, T40 and then under the influence of SM $+ T$40, INH $+ T$40, PAS $+ T$40. A strain of fast-growing *Myc.* 607 was used for the investigations.

Experimental technique. Into identical flasks with a specified quantity of culture medium (broth $+ 1\%$ glucose) containing no antibacterial drug (control), and into flasks with a corresponding quantity of one drug or of two drugs simultaneously, there were dropped the same quantities of bacterial suspension. The retorts were placed in an incubator at a temperature of 37°C.

The concentrations of the substances in the flasks were established on the basis of a few determinations of the resistance of the investigated strain in test-tubes.

After the growth in flasks had been noted at weekly intervals, a suspension in a bacteriological mortar was prepared out of bacilli growing in the flasks in the form of a layer on the culture medium surface. It was dropped into test-tubes with culture medium containing different concentrations of a particular drug, and into test-tubes containing no drug (control for strain vitality).

The bacilli growing in a retort containing two drugs, e.g. SM $+ T$40, were examined as to their resistance against SM and T40. Every time this was done the bacteria growing in flasks containing no drugs were simultaneously examined as to their drug resistance, for control purposes.

The above experiments gave the following results:

1. Bacilli of *Myc.* 607 strain, grown in the presence of SM only and possessing an initial resistance of 1.25 μg./ml., after three weeks' culti-

Note. T40 — bromosalicylhydroxamic acid.

vation, became resistant against 50,000 μg. SM/ml., i.e. about 40,000 times more resistant than at the beginning of the experiment. During the next two weeks of cultivation the resistance diminished, probably on account of the decreased vitality of the strain.

After 6 weeks no growth was obtained.

2. When bacilli were cultivated in the presence of SM + T40 it was found that resistance against SM was lowered and deferred in its appearance. Resistance was highest after a 4 weeks' incubation period and was equal to 25,000 μg. SM/ml., hence it was about 20,000 times higher than at the beginning of the test and half as high as in the case of SM alone.

3. Bacilli cultivated in the presence of INH became resistant against 125 μg. INH/ml. in 3 weeks; after 4 and 5 weeks, in consequence of their lowered vitality, they grew in the presence of 62.5 μg. INH/ml.; after 6 weeks, growth was hardly obtained in 12.6 μg. INH/ml.; after 7 weeks, no growth was observed.

4. Bacilli cultivated in the presence of INH + T40 did not show any increased resistance against INH after a 5 weeks' incubation period.

Bacilli cultivated in the presence of PAS, and in that of PAS + T40 did not change their resistance during the whole experimental period.

6. Bacilli cultivated in the presence of T40 for 5 weeks did not change their resistance against T40 (150 μg./ml.). After 6 weeks' incubation a lower bacilli vitality was observed, they grew in 75 μg/ml. After 7 weeks' incubation the bacilli were unable to propagate.

7. Bacilli cultivated in the presence of T40 + PAS behaved in a similar way.

8. Bacilli cultivated in media containing T40 + SM or T40 + INH were resistant against 150 μg. T40/ml. during 3 weeks of incubation. After 4 weeks, they grew in 75 μg. T40/ml., but after 5 weeks no growth was found in passages.

The investigations were carried out with a fast-growing strain which, though described by Gottlieb, Legator and Bevan as *Myc. tuberculosis* 607, revealed in tests carried out *in vitro* and *in vivo* in our laboratory the properties of a saprophitic strain. Moreover, it must be remembered that no far-reaching conclusions for clinics can be drawn on the basis of laboratory tests, especially in view of the fact that in previous papers on the combined effect of chemotherapeutics upon mycobacterial strains, different results were obtained when different strains were used.
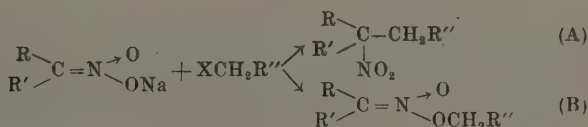
TUBERCULOSIS INSTITUTE, WARSAW.

On the Alkylation of Derivatives
of 5-Nitro-1,3-dioxane

by

Z. ECKSTEIN and T. URBAŃSKI

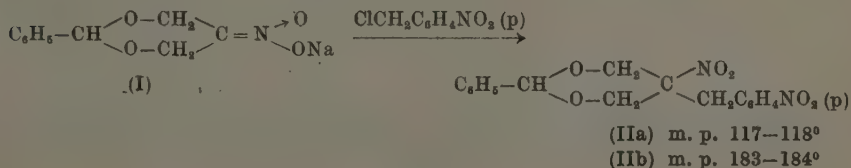*Communicated by T. URBAŃSKI at the meeting of May 16, 1955*

It is a well-known fact that the action of some aralkyl chlorides on sodium salts of nitroparaffins produce aralkyl derivatives of nitro-paraffins (A), through the formation of a new C-C bond: —
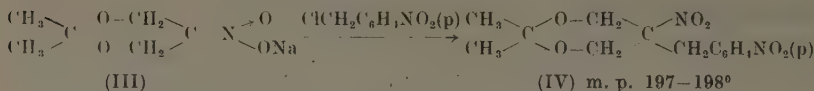
$$\begin{array}{c} R \\ R' \end{array}\!\!C = N \!\!\begin{array}{c} \to O \\ ONa \end{array} + XCH_2R'' \Big\langle \begin{array}{c} \nearrow \begin{array}{c} R \\ R' \end{array}\!\!C - CH_2R'' \\ \qquad \underset{NO_2}{|} \\ \searrow \begin{array}{c} R \\ R' \end{array}\!\!C = N \!\!\begin{array}{c} \to O \\ OCH_2R'' \end{array} \end{array} \qquad \begin{array}{c} (A) \\ \\ (B) \end{array}$$

In addition to the (A) compounds, i.e. C-aralkyl derivatives, there are also formed O-aralkyl derivatives (B). They are unstable and decompose readily.
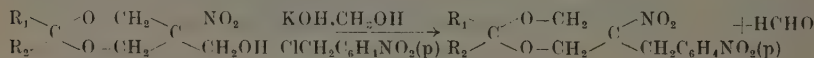
The reaction can be carried out when R'' is o-nitro-[1], p-nitro-[2]-[4], or 2,4-dinitrophenyl[5]. According to [6], the same reaction can be carried out between silver salt of nitroparaffins and trityl chloride.

The authors of the present paper acted on sodium salt of 2-phenyl--5-nitro-1,3-dioxane (I) or 2,2-dimethyl-5-nitro-1,3-dioxane (III) with p-nitrobenzyl chloride in anhydrous methanol so as to yield compounds (II) and (IV) respectively. Compound (II) exists in two isomeric forms: (IIa) — m.p. 117—118° and (IIb) — m.p. 183—184°. They are probably stereoisomers: the existence of stereoisomers of 5-nitro-5-alkyl-1,3-dioxanes has already been shown by Senkus and other workers [7], [8].
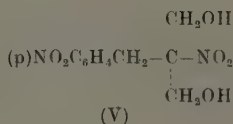
$$C_6H_5-CH\!\!\begin{array}{c} \diagup O-CH_2 \\ \diagdown O-CH_2 \end{array}\!\!C = N\!\!\begin{array}{c} \to O \\ \diagdown ONa \end{array} \xrightarrow{\;ClCH_2C_6H_4NO_2\,(p)\;}$$

(I)

$$C_6H_5-CH\!\!\begin{array}{c} \diagup O-CH_2 \\ \diagdown O-CH_2 \end{array}\!\!C\!\!\begin{array}{c} \diagup NO_2 \\ \diagdown CH_2C_6H_4NO_2\,(p) \end{array}$$

(IIa) m. p. 117—118°
(IIb) m. p. 183—184°

$$\underset{\text{(III)}}{\overset{\text{CH}_3}{\underset{\text{CH}_3}{\diagdown}}\text{C}\overset{\text{O}-\text{CH}_2}{\underset{\text{O}-\text{CH}_2}{\diagdown}}\text{C}\overset{\diagup\text{O}}{\underset{\diagdown\text{ONa}}{N}}} \quad \xrightarrow{\text{ClCH}_2\text{C}_6\text{H}_4\text{NO}_2(\text{p})} \quad \underset{\text{(IV) m. p. }197-198^0}{\overset{\text{CH}_3}{\underset{\text{CH}_3}{\diagdown}}\text{C}\overset{\diagup\text{O}-\text{CH}_2}{\underset{\diagdown\text{O}-\text{CH}_2}{\diagdown}}\text{C}\overset{\diagup\text{NO}_2}{\underset{\diagdown\text{CH}_2\text{C}_6\text{H}_4\text{NO}_2(\text{p})}{}}}$$

The same products (II) and (IV) were prepared, using instead of (I) and (III) respectively, their 5-hydroxymethyl derivatives, in the presence of KOH. This was evidently due to the splitting off of formaldehyde in the alkaline medium:

$$\underset{\text{R}_2}{\overset{\text{R}_1}{\diagdown}}\text{C}\overset{\diagup\text{O}-\text{CH}_2}{\underset{\diagdown\text{O}-\text{CH}_2}{\diagdown}}\text{C}\overset{\diagup\text{NO}_2}{\underset{\diagdown\text{CH}_2\text{OH}}{}} \quad \xrightarrow[\text{ClCH}_2\text{C}_6\text{H}_4\text{NO}_2(\text{p})]{\text{KOH.CH}_2\text{OH}} \quad \underset{\text{R}_2}{\overset{\text{R}_1}{\diagdown}}\text{C}\overset{\diagup\text{O}-\text{CH}_2}{\underset{\diagdown\text{O}-\text{CH}_2}{\diagdown}}\text{C}\overset{\diagup\text{NO}_2}{\underset{\diagdown\text{CH}_2\text{C}_6\text{H}_4\text{NO}_2(\text{p})}{}} \quad +\text{HCHO}$$

The isomeric substances (II), when hydrolised with hydrochloric acid yielded the same 2-nitro-2-(p-nitrobenzyl)-1,3-propanediol (V):

$$\begin{array}{c} \text{CH}_2\text{OH} \\ | \\ (\text{p})\text{NO}_2\text{C}_6\text{H}_4\text{CH}_2-\text{C}-\text{NO}_2 \\ | \\ \text{CH}_2\text{OH} \end{array}$$
$$\text{(V)}$$

The same product was obtained by hydrolysis of (IV). The diol (V) prepared from (IIa) or (IIb) when subjected to cyclisation with acetone produced the ketal, identical with (IV). When diol (V) prepared from (IV) was subjected to cyclisation with benzaldehyde, the acetal (II) thus obtained was identical with (IIb), i.e. with the higher melting isomer.

Diol (V) reacted with acetic anhydride or p-nitrobenzoyl chloride in the presence of pyridine, yielding the corresponding esters.

When diol (V) in alcoholic solution was warmed to $60-75^0$ with benzylamine and formaldehyde in the molar ratio $1:1:2$, there resulted 5-nitro-5-(p-nitrobenzyl)-N-benzyl-tetrahydro-1,3-oxazine (VI) m.p. $138.5-139^0$ with a good yield (c. 80% of the theoretical). When hydrolised with hydrochloric acid diluted with alcohol, it furnished hydrochloride of 2-nitro-2-(p-nitrobenzyl)-3-(N-benzyl)-propan-1-ol-3-amine (VII) m. p. $152-153^0$. Simultaneously, one mol of formaldehyde was split off:

$$\underset{\text{(VI)}}{\begin{array}{c} \text{NO}_2\diagdown_{\text{C}}\diagup\text{CH}_2\text{C}_6\text{H}_4\text{NO}_2(\text{p}) \\ \text{CH}_2 \quad \text{CH}_2 \\ | \qquad | \\ \text{O} \quad \text{N}-\text{CH}_2\text{C}_6\text{H}_5 \\ \diagdown_{\text{CH}_2}\diagup \end{array}} \quad \xrightarrow[+\text{H}_2\text{O}]{\text{H}^+} \quad \underset{\text{(VII)}}{\begin{array}{c} \text{NO}_2\diagdown_{\text{C}}\diagup\text{CH}_2\text{C}_6\text{H}_4\text{NO}_2(\text{p}) \\ \text{CH}_2 \quad \text{CH}_2 \\ | \qquad | \\ \text{OH} \quad \text{NH}_2\text{CH}_2\text{C}_6\text{H}_5 \\ \qquad \text{Cl}^- \end{array}} +\text{CH}_2\text{O}$$
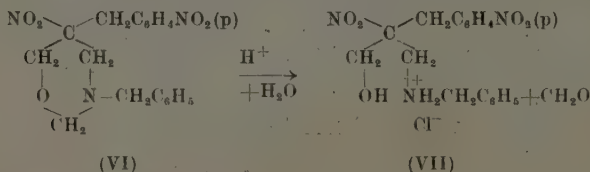
Substance (VII) in the form of a free base, when warmed with formaldehyde, yielded product (VI).

The reactions described above constitute a new method of preparing derivatives of ω-nitroethylbenzene. The new products obtained by the authors, together with their melting points, are tabulated below.

Name	Number	m. p.
5-nitro-5-(p-nitrobenzyl)-2-phenyl-1,3-dioxane	(IIa)	$117-118^0$
" " " "	(IIb)	$183-184^0$
5-nitro-5-(p-nitrobenzyl)-2,2-dimethyl-1,3-dioxane	(IV)	$197-198^0$
2-nitro-2-(p-nitrobenzyl)-1,3-propandiol	(V)	$86- 87^0$
diacetyl derivative of (V)	—	$93-94.5^0$
di(p-nitrobenzoyl) derivative of (V)	—	$187-188^0$
5-nitro-5-(p-nitrobenzyl)-N-benzyl-tetrahydro-1,3-oxazine	(VI)	$138.5-139^0$
2-nitro-2-(p-nitrobenzyl)-3-(N-benzyl)-propan-1-ol- -3-amine hydrochloride	(VII)	$152-153^0$
free base of (VII)	—	$112-114^0$

INSTITUTE OF ORGANIC SYNTHESIS, POLISH ACADEMY OF SCIENCES,

DEPARTMENT OF ORGANIC TECHNOLOGY, INSTITUTE OF TECHNOLOGY, WARSAW

REFERENCES

[1] T. Posner, Ber. **31** (1898), 656; Ann. **389** (1912), 1.
[2] H. B. Hass, M. L. Bender, J. Am. Chem. Soc., **71** (1949), 1767;
[3] H. B. Hass, E. J. Berry, M. L. Bender, J. Am. Chem. Soc., **71** (1949), 2290;
[4] T. W. Hoover, H. B. Hass, J. Org. Chem., **12** (1947), 501.
[5] L. Weilser, R. W. Helmkamp, J. Am. Chem. Soc., **67** (1945), 1167.
[6] H. Wieland, A. Höchtlen, Ann. **505** (1933), 237.
[7] M. Senkus, J. Am. Chem. Soc., **65** (1943), 1656;
[8] A. Scattergood, A. L. MacLean, J. Am. Chem. Soc., **71** (1949), 4153.

On the Products of the Reaction of 1-Nitropropane with Formaldehyde and Ethylenediamine

by

R. KOLIŃSKI and T. URBAŃSKI

Communicated by T. URBAŃSKI at the meeting of May 16, 1955

Continuing their former experiments on the reactions of primary nitro-paraffins with formaldehyde and ammonia or amines [1]-[3], the authors now examined the products of the reaction of 1-nitropropane with formaldehyde and ethylenediamine in the molar ratio 2:4:1. When the components were mixed at room temperature a spontaneous reation occurred and the resinous product of reaction was precipitated. It was then treated in the following way.

The resin was dissolved in ethanol and the solution was left to undergo slow evaporation. This yielded a crystalline amine (m.p. 104-106°) to which we ascribed the structure — 3,7-dinitro-3,7-diethyl-1,5-diazabicyclo (3,3,2) decane (I). The residue from the crystallisation was a resinous polymer which was not investigated further.

The following were the properties of the amine (I).

Under the action of alcoholic hydrogen chloride, it formed a hydrochloride of m.p. 140-141°. The hydrochloride of (I) was readily hydrolised by water and upon dissolving in water the free amine (I) precipitated.

The amine (I) did not react with methyl iodide. It was stable in the presence of cold conc. hydrochloric acid and cold 10% NaOH, but decomposed upon being heated.

When the hydrochloride of (I) was refluxed for 2 hours in ethanol a certain amount of the base (I) was subjected to a partial hydrolysis, the formaldehyde was split off and hydrochloride of 1-(2-nitro-*n*-butyl)--6-nitro-6-ethyl-1,4-diazacycloheptane (II) (m.p. 160-162°) was formed. Under the action of nitrous acid product (II) yielded an oily N-nitroso-compound. Further warming of hydrochloride (II) or the N-nitrosoderivative with conc. hydrochloric acid yielded hydrochloride of 6-nitro-6--ethyl-1,4-diazacycloheptane (III) decomp. c. 300°. The other products of reaction were: formaldehyde and a primary nitroparaffin, presumably

1-nitropropane. Under the action of nitrous acid the base (III) yielded an N-dinitrosoderivative (m.p. 124°). Under the action of tosyl chloride it yielded a ditosyl derivative (m.p. 169°).

The ultraviolet absorption spectra of (I) and (II) show a maximum of c. 260 mμ., which is specific for the nitro group. The base (III) does not show a maximum, only a bend of c. 250 mμ. which, according to the hypothesis of one of us [4][5], is produced by the hydrogen bond between the oxygen atom of the NO_2 group and the hydrogen of the NH group (IIIa) forming a six-membered chelate ring.

A similar chelate ring can be formed between one of the nitrogroups and the hydrogen of the -NH- group of (II). Another nitrogroup of (II) remains "unbound" and suffices to give a maximum of c. 260 mμ. (IIa).

This constitutes an additional proof of the structures ascribed to products (I), (II) and (III).
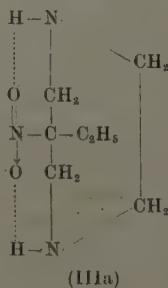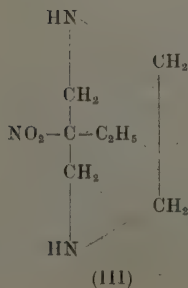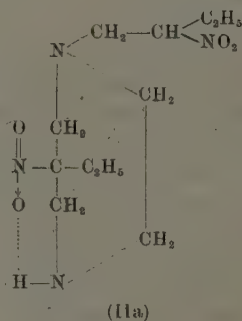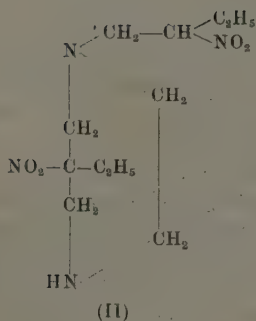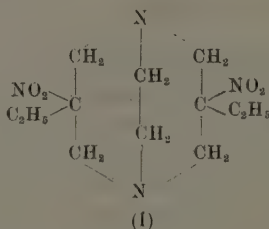
A further proof was obtained by a partial synthesis of (I): when amine (III) was warmed with 2-nitro-2-ethyl-propanediol-1,3 (IV) in the presence of aqueous sodium hydroxide, base (I) was obtained:

$$
\begin{array}{c}
\text{(III)} \qquad\qquad \text{(IV)}
\end{array}
$$
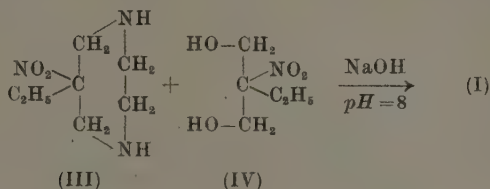
Experiments to synthesise the ditosyl derivative of (III) in a way analogous to the known synthesis of 1,4-diazacycloheptane [6]-[8], starting form N,N′-ditosyl-ethylenediamine were unsuccessful.

A new intermediate was obtained to carry out this synthesis. This was 1,3-dichlor-2-nitro-2-ethyl-propane (V) (m.p. 48-50°). It was prepared from diol (IV) by an action of thionyl chloride in pyridine, in a way analogous to the known method of preparing trichloromethylnitromethane [4]. A by-product — sulphurous ester of 2-nitro-2-ethyl-propanediol-1,3 (VI) (m.p. 68-70°) was also obtained.

$$
\begin{array}{cc}
\text{(V)} & \text{(VI)}
\end{array}
$$
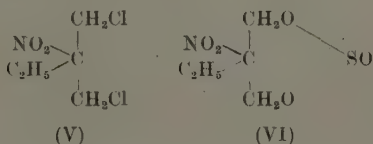
INSTITUTE OF ORGANIC SYNTHESIS, POLISH ACADEMY OF SCIENCES.

REFERENCES

[1] E. L. Hirst, J. K. N. Jones, S. Minahan, F. W. Ochyński, A. T. Thomas, T. Urbański, J. Chem. Soc. (1947), 924.

[2] T. Urbański, E. Lipska, Roczniki Chem. **26** (1952), 182.

[3] T. Urbański, D. Gürne, Roczniki Chem. **28** (1954), 175.

[4] T. Urbański, Bull. Acad. Polon. Sci.. Cl. III, **1** (1953), 239; **2** (1954), 394.

[5] T. Urbański, D. Ciecierska, Roczniki Chem. **29** (1955), 11.

[6] L. Bleier, Ber., **32** (1899), 1825.

[7] C. Howard, W. Marckwald, Ber. **32** (1899), 2038.

[8] S. M. McElvain, L. W. Bannister, J. Am. Chem. Soc. **76** (1954), 1126.

[9] G. Fort, A. McLean, J. Chem. Soc. (1948), 1902.

The Catalytic Activity and Electric Conductivity of MgO-Cr$_2$O$_3$ Mixed Catalysts

by

A. BIELAŃSKI, J. DEREŃ and J. HABER

Presented by B. KAMIEŃSKI on July 5, 1955

In a previous paper [1] we proved the existence of a distinct parallelism between the changes in electric conductivity of ZnO-Fe$_2$O$_3$ mixtures working as catalysts in the reaction of dehydrogenation of C$_2$H$_5$OH and the yield of catalytic reaction. After the vapours of ethyl alcohol — water mixture had been introduced into the reaction chamber, the electric conductivity of the catalyst increased; the greater the increase of the conductivity, the greater was the catalytic reaction yield. In a brief note [2] recently published, similar results were obtained by Boreskoff and Matweeff who investigated the oxidation of methyl alcohol on ZnO as catalyst and by Psheshesky and Miasnikoff, for the dehydrogenation of isopropyl alcohol on the same catalyst.

The investigations of Lashkarieff and his school [3] showed that there also exists a relation between the electronic work function and the electric conductivity changes of semi-conducting materials taking place in the course of chemisorption. In the case of normal semi-conductors (electronic conduction), the change in the conductivity shows a sign opposite to that of abnormal semi-conductors (positive hole conduction). The catalyst applied in our previous investigation [1] represented a mixture of two normal semi-conductors. In the present research, we investigated ethyl alcohol dehydrogenation using as catalyst a mixture of MgO and Cr$_2$O$_3$, both oxides being abnormal semi-conductors.

We applied catalysts composed as follows: 4MgO+Cr$_2$O$_3$, 3MgO+ +Cr$_2$O$_3$, 2MgO+Cr$_2$O$_3$, MgO+Cr$_2$O$_3$, MgO+2Cr$_2$O$_3$, MgO+3Cr$_2$O$_3$ and MgO+4Cr$_2$O$_3$ as well as pure Cr$_2$O$_3$. The resistance of the MgO samples was too high to be measured with our apparatus. We carried out our measurements in the same way and with the help of the same apparatus as previously. The measurements of the conductivity of the catalyst and of the catalytic reaction yield were carried out within a tem-

perature range of 350 up to 650°C. In all the measurements the same reactant was used, i.e. 20%/₀ ethyl alcohol-water mixture.

The curves in Fig. 1 show the change in the logarithm of the conductivity of the catalyst (equimolar mixture $MgO + Cr_2O_3$) taking place after the reacting vapours were introduced into the reaction chamber. At a temperature of 350°C (curve I) and also at 400° and 500° the conductivity reaches after a few minutes, a constant value lower than the initial conductivity before the reaction. The sign of the conductivity change in this case is therefore opposite to that of the previous case, when the normal semi-conducting oxides ZnO and Fe_2O_3 were used. At temperatures of 550°, 600° and 650°C the log σ — time curves show a more complicated form. At the very beginning the conductivity decreases; but after some time (the shorter the time, the higher the temperature) it again increases up to a level higher than the initial level. This behaviour is represented by curve II in Fig. 1, taken for the temperature of 650°C. In all cases, we shall denote by $\Delta \log \sigma_1$, the value corresponding to a decrease in the conductivity and by $\Delta \log \sigma_2$, the value corresponding to an increase from the lowest to the highest conductivity of the sample as shown by the arrow in Fig. 1.
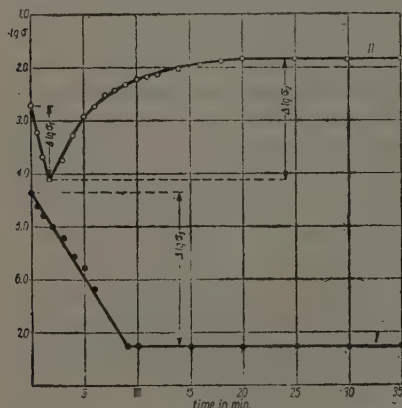
Fig. 1. The change in log σ of the catalyst in the course of the reaction of dehydrogenation of ethyl alcohol vapours for the $MgO + Cr_2O_3$ mixture.

In the case of catalysts of the compositions $4MgO + Cr_2O_3$, $3MgO + Cr_2O_3$, $2MgO + Cr_2O_3$, we observed at all temperatures a decrease in the conductivity only (log σ — time curves analogous to curve I). In the case of catalysts of the compositions $MgO + 2Cr_2O_3$, $MgO + 3Cr_2O_3$, $MgO + 4Cr_2O_3$, as well as in the case of pure Cr_2O_3, we observed for temperatures lower than $450 - 500°$ log σ — time curves similar to curve I, and for higher temperatures curves similar to curve II. In all cases after the reacting vapours were pumped off and air let into the apparatus at the initial pressure (10 mm Hg), the conductivity again reached its initial value.

It seems that the changes taking place in the behaviour of Cr_2O_3 rich catalysts at about 500° are connected with a polymorphic change in Cr_2O_3. The temperature of this change depends largely on the previous history of the sample. Wohlen and Wernung [4] found that for various samples it changed from 420° up to 630°. The samples of Cr_2O_3 used in

the present investigation showed a distinct break in the $\log \sigma - \dfrac{1}{T}$ curve at a temperature of about 500°C. when the conductivity was measured in air. Such a break was also observed with Cr_2O_3-MgO mixtures containing one or more mols of Cr_2O_3 to one mol of MgO. No break was observed in the course of measurements of the conductivity in air with the samples with a lower Cr_2O_3 content.

Parallel to the measurements of the electric conductivity of the catalyst the yield of the catalytic reaction was determined. According to Dolgoff [6], the catalytic oxidation of ethyl alcohol into acetone passes through a number of intermediate products: acetaldehyde, acetic acid and ethyl acetate. In our apparatus we obtained, owing to the small quantity of the catalyst and the short period of its contact with the reagents, only traces of acetone. The reaction products consisted mainly
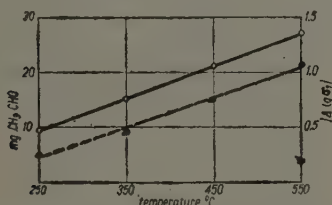
Fig. 2. The change of the $|\varDelta \log \sigma_1|$ value for $3MgO + Cr_2O_3$ mixture (full line) and the catalytic reaction yield (dotted line), in both cases as a function of the temperature of the reaction
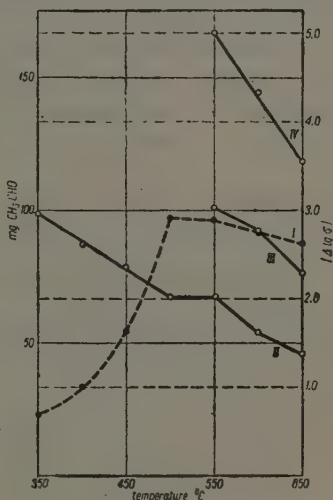
Fig. 3. The change in $|\varDelta \log \sigma_1|$, $|\varDelta \log \sigma_2|$ and $|\varDelta \log \sigma_1| + |\varDelta \log \sigma_2|$ values for $MgO + Cr_2O_3$ mixture (full line) and the catalytic reaction yield (dotted line), in both cases as a function of the reaction temperature

of acetaldehyde with small quantities of acetic acid. After passing 20 ml of 20% alcohol-water mixture through the apparatus, the joint content of CH_3CHO and CH_3COOH was determined with hydroxylamine hydrochloride in the condensation products and expressed in mg of CH_3CHO. This value was subsequently taken as the measure of the reaction yield.

Fig. 2 shows the absolute value of $\varDelta \log \sigma_1$ as a function of the reaction temperature (full line) and also the dependence of the reaction yield on the temperature (dotted line). The measurement shown in Fig. 2 was carried out with a catalyst of the composition $3MgO + Cr_2O_3$. The shape of the analogous curves obtained with the mixtures $4MgO +$

$+Cr_2O_3$ and $2MgO+Cr_2O_3$ as catalysts was similar, i.e. the larger $|\Delta \log \sigma_1|$ the larger the reaction yield. In the temperature range observed the dependence of both values may be taken in the first approximation as linear.

With samples of the compositions $MgO+Cr_2O_3$, $MgO+2Cr_2O_3$, $MgO+$ $+3Cr_2O_3$ and $MgO+4Cr_2O_3$ showing during the catalytic reaction a decrease in the conductivity at lower temperatures and a decrease and increase at higher temperatures, the curves analogous to the curves shown in Fig. 2 were much more complicated. As a typical example of the behaviour of this group of our samples we show in Fig. 3 the results of the measurements carried out with the $MgO+Cr_2O_3$ mixture. The dotted line (curve I) shows the yield of the catalytic reaction as a function of the temperature of the reaction; it increases up to a maximum at 500°C
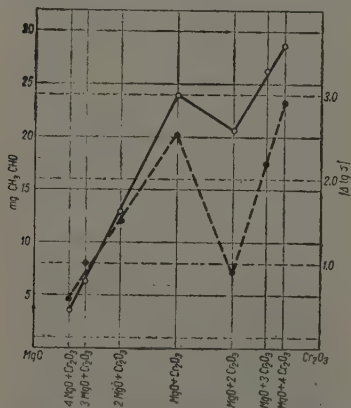
Fig. 4. The dependence of catalytic reaction yield (dotted line) and of $|\Delta \log \sigma_1|$ values on the composition of the sample. Reaction temperature 350°C
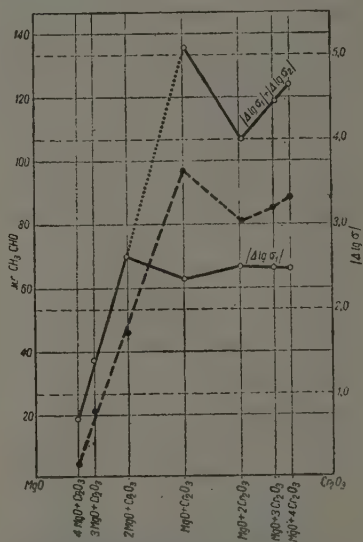

Fig. 5. The dependence of catalytic yield (dotted line) and of $|\Delta \log \sigma_1|$ as well of $|\Delta \log \sigma_1|+|\Delta \log \sigma_2|$ values (full lines) on the composition of the sample. Reaction temperature 550°C

and then decreases. The full line (curve II) shows the absolute value of $\Delta \log \sigma_1$. Curve III drawn for temperatures above 500° gives the absolute value of $\Delta \log \sigma_2$ and curve IV the sum of $|\Delta \log \sigma_1|$ and $|\Delta \log \sigma_2|$. Above 500° all the three values: $|\Delta \log \sigma_1|$, $|\Delta \log \sigma_2|$ and $|\Delta \log \sigma_1|+$ $+|\Delta \log \sigma_2|$ run parallel to the catalytic reaction yield. Below 500° the increase in the reaction yield is accompanied by a decrease in $|\Delta \log \sigma_1|$ value.

The dotted lines in Figs. 4 and 5 show the dependence of the catalytic reaction yield as measured at 350° or 550° respectively on the compo-

sition of the sample. It is low in the samples rich in MgO and increases up to a maximum for the $MgO + Cr_2O_3$ composition, then again decreases to a minimum for the $MgO + 2Cr_2O_3$ composition and once more increases. The full line in Fig. 4 shows the $|\Delta \log \sigma_1|$ value, the changes in which are parallel to the reaction yield. Fig. 5 shows that at 550° the $|\Delta \log \sigma_1|$ for MgO rich samples is also parallel to the reaction yield, and for Cr_2O_3 rich samples it is almost constant. For Cr_2O_3 rich catalysts, however, a parallel is observed between the reaction yield and $|\Delta \log \sigma_1| + |\Delta \log \sigma_2|$ values. It is of interest that the point representing $|\Delta \log \sigma_1| + |\Delta \log \sigma_2|$ for $MgO + Cr_2O_3$ mixture lies on the same line as the points corresponding to the increasing portion of the $|\Delta \log \sigma_1|$ curve.

SCHOOL OF MINING AND METALLURGY, CRACOW
DEPARTMENT OF INORGANIC CHEMISTRY.

REFERENCES

[1] A. Bielański, J. Dereń and J. Haber, *The electric conductivity and catalytic activity of ZnO-Fe₂O₃ mixtures*. Bull. Acad. Polon. Sci., Cl. III, **3** (1955), 223.

[2] Uspekhi Fiz. Nauk **55** (1955), 111.

[3] W. I. Liashenko, I. I. Stiepko, Izvest. Akad. Nauk SSSR (ser. fiz.) **16** (1952), 211.

[4] R. Abegg, *Handbuch der anorganischen Chemie*, Leipzig 1927, vol. 4, 119.

[5] B. N. Dolgoff, B. A. Bolotoff, L. A. Komissarowa, Shur. Priklad. Khim. **28** (1955), 71.

GEOLOGY

Stratigraphy of the Jurassic and Cretaceous of Bachowice

by

M. KSIĄŻKIEWICZ

Communicated at the meeting of June 20, 1955

In Bachowice, ten kilometres to the north of Wadowice, (Western Carpathians) there are beds of greenish and greyish-green marly clays with Palaeocene or Lower Eocene micro-fauna. They rest on top of greenish Upper Senonian marls belonging to the Sub-Silesian nappe. The marly clays are mixed with dark or red clays and weathered tuffs. In these beds there are great quantities of usually unrounded exotic blocks which have already been known to E. Tietze [1]. Among these, blocks of Jurassic and Cretaceous age have been identified, and the fauna they contain has made it possible to establish the age of the rocks fairly accurately. The geology of the area surrounding Bachowice has been recently described in detail by the author [2].

The Jurassic series is represented by Aalenian, Bajocian, Bathonian, Callovian, Oxfordian, Kimmeridgian, and Tithonian stages. It seems probable that the exotic blocks of Bachowice represent an almost complete Jurassic series, from the Aalenian to the Tithonian.

The Aalenian or the Early Bajocian deposits are represented by dark sandstone with ferruginous ooids. The fauna is very scarce and consists of *Varamussium pumilum* (Lam.) and *Entolium spathulatum* (Roem.)

The Bajocian is represented by dark grey or yellowish calcareous sandstones with a very rich fauna:

Dorsetensia romani (Opp.)	*Entolium spathulatum (Roem.)*
Dorsetensia aff. pulchra S. Buckm.	*Entolium disciforme (Schubler)*
Witchellia cf. eduardiana (d'Orb.)	*Lima (Radula) duplicata Sow.*
Witchellia propinquans Bayle	*Gervillia acuta Sow.*
Pseudomonotis echinata (Sow.)	*Ostrea cf. obscura Sow.*
Avicula (Oxytoma) scarburgensis Rollier	*Ostrea cf. planata Qu.*
Avicula (Oxytoma) münsteri Bronn	*Ostrea cf. calceola Ziet.*
Avicula (Oxytoma) hersilia d'Orb.	*Modiola greppini Rollier*
Avicula (Oxytoma) cf. interlaevigata Qu.	*Cucullaea subdecussata Mstr.*
Camptonectes lens (Sow.)	*Cucullaea concinna Phill.*
Varamussium pumilum (Lam.)	*Cucullaea cf. aalensis Qu.*

Trigonia (Clavotrigonia) signata Ag.
Trigonia (Clavotrigonia) goldfussi Ag.
Trigonia (Clavotrigonia) clavellulata Str.
Trigonia (Clavotrigonia) cf. producta Lyc.
Trigonia (Clavotrigonia) cf. spinulosa Y.
  et B.
Trigonia (Lyriodon) cf. hemisphaerica Lyc.
Trigonia (Lyriodon) cf. denticulata Ag.
Astarte cf. voltzii Hoen.
Astarte pulla Roem.
Astarte depressa Mstr.

Astarte minima Phill.
Astarte exarata Koch et Dunker
Lucina bellona d'Orb.
Lucina zonaria Qu.
Tancredia donaciformis Lyc.
Cardium cf. concinnum Mor. Lyc.
Cardium cf. incertum Phill.
Pholadomya murchisoni Sow.
Pleuromya elongata Mstr.
Acanthothyris cf. spinosa (Schloth.)

The fauna covered by the list is representative of both the Lower and the Upper Bajocian. The best represented are the sowerbyi, romani and humphriesi zones. It is possible that some of the sandstones date from the Early Bathonian.

It seems that the Bathonian is mainly represented by pink or grey marls containing in profusion the forms *Posidonomya alpina* Gras. besides *Calliphylloceras disputabile* (Zitt.) and some other unidentifiable Phylloceratidae. To the same stage belong the variegated limestones which are very common in Bachowice. They are pink or yellowish with green or red bandings, and may also represent the Callovian. The fauna found in these limestones is scarce and consists of:

Oppelia cf. costata Roem.
Perisphinctes (Grossouvreia) cf. curvicosta
  (Opp.)
Sowerbyceras cf. subtortisulcatum Pomp.
Phylloceras sp.
Rhynchonella trigonella Rothpletz
Rhynchonella cf. kamiński Uhl.

Rhynchonella sp. ind.
Terebratula sp.
cf. Neritopsis haueri Uhl.
Crossostoma sp.
Turbo sp.
Fragments of crustaceans, snails, and fish teeth.

Of Callovian or perhaps Early Oxfordian age are red crinoidal limestones, which occur in some blocks as intercalations together with the variegated limestones. The fauna in the crinoidal limestone consists of such Callovian and Lower Oxfordian forms as *Rhynchonella orbignyana* Opp., *Terebratula haasi* Rollier, and unidentified *Phylloceratidae.*

The Oxfordian stage – sensu lato – is represented by crinoidal limestones, pink, bluish, greenish, or yellowish in colour. All the four varieties of the limestone contain a fairly rich and similar fauna which indicates that the Peltoceras transversarium zone is best represented. However, it seems likely that these limestones represent also the Lower and the Upper Oxfordian zones.

The following forms have been identified:

Calliphylloceras zignoi (d'Orb.)
Calliphylloceras manfredi (d'Orb.)
Calliphylloceras disputabile (Zitt.)
Phylloceras douvillei Choffat
Sowerbyceras tortisulcatum (d'Orb.)

Thysannolytoceras cf. adeloides (Kud.)
Taramelliceras kobyi (Choffat)
Taramelliceras pichleri (Opp.)
Taramelliceras sarasini (Lor.)
Taramelliceras (Proscaphites) anar (Opp.)

Trimarginites trimarginatus (Opp.)
Cardioceras aff. cordatum (Sow.)
Cardioceras tenuiserratum (Opp.)
Perisphinctes (Properisphinctes) bernensis Lor.
Perisphinctes jelskii Siem.

Perisphinctes orbignyi Lor.
Perisphinctes cf. delgadoi Choffat
Perisphinctes (Discosphinctes) lucingense Favre
Peltoceras neumayri Jeannet
Euaspidoceras cf. perarmatum (Sow.)

The Kimmeridgian is represented by greenish pelitic limestone containing an abundant fauna which is composed of:

Lytoceras polycyclum Neum.
Lytoceras cf. orsini Gem.
Sowerbyceras tortisulcatum (d'Orb.)
Haploceras tenuifalcatum Neum.
Metahaploceras strombecki (Opp.)
Taramelliceras greenackeri (Moesch.)
Taramelliceras cf. trachynotum (Opp.)

Taramelliceras pseudoflexuosum (Farre)
Idoceras cf. sautieri (Font.)
Ataxioceras cf. stromeri Wegele
Physodyceras uhlandi (Opp.)
Avicula lacunosa Qu.
Rhynchonella cf. spoliata Suess

From the above list it seems likely that the greenish limestones represent entirely the Kimmeridgian stage, as the species which are found in them are representative both of the lower *(Ataxioceras stromeri)* and the upper *(Idoceras sautieri)* zones. It may be the greyish limestones from which no fossils have been identified are also of Kimmeridgian age, as their structure in thin plates is similar.

In Bachowice the Tithonian is represented by white sub-crystalline limestones with abundant ammonites or by Calpionella limestones.

The fauna in the sub-crystalline limestone consists of the following forms:

Thysannolytoceras cf. montanum (Opp.)
Thysannolytoceras cf. liebiegi (Opp.)
Protetragonites quadrisulcatum (d'Orb.)
Lytoceras sp.
Calliphylloceras cf. calypso (d'Orb.)
Phylloceras sp.
Sowerbyceras cf. tortisulcatum d'Orb.
Haploceras elimatum (Opp.)

Aulacostephanus sp. cf. pachygyrus Uhl.
Perisphinctes sp. (s. l.)
Lamellaptychus beyrichi (Opp.)
Astarte sp.
Pygope diphya (Col.)
Glossothyris bouei Zejszn.
Rhynchonella cf. capillata Zitt.
Rhynchonella suessi Zitt.

The fauna in the white or yellowish Calpionella limestone consists of:

Lytoceras sp.
Phylloceras sp.
Calliphylloceras calypso (d'Orb.)
Calliphylloceras (Ptychophylloceras) cf. semisulcatum (d'Orb.)
Haploceras sp.
Berriasella cf. oppeli Kilian
Belemnites sp.
Rhynchonella cf. zeuschneri Zitt.

Rhynchonella sparsicosta Qu.
Rhynchonella aliformis Rollier
Glossothyris cf. bouei (Zejszn.)
Terebratula carpathica Zitt.
Terebratula bieskidensis Zejszn.
Waldheimia (Zeilleria) cf. humeralis (Roem.)
Calpionella alpina Lor.
Calpionella elliptica Cadish

White limestones, similar to the one just described but containing abundant crinoids, are also Tithonian. The forms found here are:

Sowerbyceras cf. tortisulcatum (d'Orb.)
Taramelliceras sp. ind. (cf. otreropleurus Font.)
Taramelliceras sp.
Perisphinctes sp. (s. l. ind.)

Belemnites sp.
Rhynchonella capillata Zitt.
Terebratula simplicissima Suess
Waldheimia (Zeilleria) cf. humeralis (Roem.)

Moreover, the grey marls containing cf. *Microcanthoceras micro-canthus* (Opp.) may also be considered as Tithonian, while the grey-brown silstones with *Lytoceras quadrisulcatum* d'Orb. are probably Early Neocomian.

The Jurassic beds of Bachowice were most probably transgressed by the Upper Cretaceous, as among the exotic blocks of the area numerous limestones with Cretaceous fauna occur. These limestones contain numerous rounded boulders and rock fragments of the Tithonian and Kimmeridgian limestones and are pink, green, red, or white in colour. Their age has been determined mainly on the basis of *Globotruncanae* which are very abundant in these limestones. These fossils make it possible to establish the age sequence of the Cretaceous limestones in Bachowice. This is as follows:

1) Pink limestone with an association of *Globotruncanae* composed of: *Globotruncana apenninica, stephani, renzi, helretica* and cf. *turonica*. This fauna is Cenomanian, probably Upper Cenomanian.

2) Pink limestone with an association of *Globotruncanae* composed of: *Gl. cf. apenninica, lapparenti angusticarinata, lapp. coronata, lapp. lapparenti, lapp. tricarinata, marginata* and *globigerinoides*. This fauna is indicative of the Lower Turonian.

3) Green limestone with an association of: *Gl. stephani, cf. turonica, lapp. lapparenti, marginata, and cf. rosetta*. This fauna is also Lower Turonian.

4) Pink limestone with *Gl. lapp. inflata, Gl. lapp. angusticarinata, lapp. coronata, lapp. lapparenti, lapp. tricarinata, marginata, and globigerinoides*. Once more the fauna is Lower Turonian.

5) Pink limestone with *Gl. cf. turonica, lapp. angusticarinata, lapp. coronata, lapp. lapparenti, lapp. tricarinata, marginata, and globigerinoides*. *Inocerami (Inoceramus cuvieri and In. inaequivalvis)* have also been identified in these limestones by F. Mitura, and they point to the late Lower Turonian age.

6) Pink limestone with *Gl. lapp. lapparenti, lapp. bulloides, lapp. tricarinata, marginata, globigerinoides, and ventricosa*. In this association the Lower Turonian forms are absent, and the limestone may therefore be considered as Upper Turonian. This is indicated also by the *Inocerami (In. cf. inconstans rotundatus)* determined by F. Mitura.

7) Green limestones with the same *Globotruncanae* association as the one just listed under 6). However, the *Inocerami* which occur here

are indicative of the Coniacien *(In. umbonatus* and *In. cf. kleini* determined by F. Mitura).

8) White limestone with *Gl. lapp. lapparenti, lapp. tricarinata, marginata, ventricosa,* and some few *Gl. arca.* It seems possible that the limestone is Upper Coniacien.

9) Brick-red limestones. In these limestones, in addition to such forms as *Gl. lapparenti, lapp. tricarinata, marginata and ventricosa* which occur in the associations already enumerated, also abundant Senonian and even Upper Senonian forms are found. To the Senonian belongs *Gl. arca, cf. fornicata,* while *Gl. cf. leupoldi* and *cf. conica* are Upper Senonian. In the sandy varieties of these limestones the micro-fauna is poor, and from the *Globotruncanae* only *Gl. cf. stuarti* and *cf. globigerinoides* appear. The brick-red limestones may therefore be regarded as of Santonian and Campanian age, and the sandy varieties are possibly Maestrichtian. The fragments of red limestone occuring in the tuffs among the exotics from Bachowice define the age of the tuffs as Campanian or even younger.

The pink, green, and white limestones contain, besides the *Globotruncanae,* abundant *Globigerinae* (inclusive of the *Globigerina cretacea* group), and even more abundantly the *"Fissurinae" (Fissurina ovalis* (Kaufm.)), while only seldom will there be a cross-cut in which *Oligostegina laevigata* (Kaufm.) may be identified. In the brick-red limestone these organisms are not found. Benthonic Foraminifera are extremely rare in all these limestones, and the Cretaceous limestones of Bachowice may be regarded as sediments composed almost entirely of planktonic organisms *(Globotruncanae, Gümbelinae, Globigerinae,* and *Fissurinae).*

The Jurassic and the Cretaceous beds of Bachowice have no lithologic equivalents in the Jurassic or Cretaceous deposits of the same age from the region of the Carpathian Mountains or their foreland. It is to be concluded, therefore, that the Jurassic beds of Bachowice represent the profile of a Jurassic which extended in the substratum of the West Carpathian Flysch basin.

The deep-neritic or bathyal character in the development of these sediments beginning from the Bathonian indicates that the deposits could not have been developed only locally, but must have extended over a considerable area in the Carpathian geosyncline. The abundance of *Phylloceratidae, Lytoceratidae,* and *Haploceratidae* shows that the sea in which the Bachowice Jurassic sediments were deposited was within the Mediterranean province. The sediments were probably deposited in a fairly deep trough fringed by reefs in the Tithonian. The coastal reefs of this trough are represented by the limestones of Kruhel and Andrychów in Poland, and Stramberk in Czechoslovakia, while the pelagic limestones of the Bachowice Tithonian were deposited in the axial zone of the trough.

After the Neocomian and before the Cenomanian the Jurassic of Bachowice emerged above sea level, and then was only partially submerged in the Upper Cretaceous. The planktonic limestones which were then deposited have no correlates within the area occupied by the Carpathian geosyncline, thovgh it may be compared in its character and micro-fauna to the limestones from the Helvetian zone of the Alps (Seewer Kalk). It seems that the sedimentation zone of the limestone from Bachowice bordered with the sedimentation zone of the variegated marls, which were deposited in the axial zone of the Flysch geosyncline. In consequence of Laramian movements the Upper Cretaceous limestone was uplifted together with the Jurassic substratum and the Palaeozoic (Dinant limestone and Productive Carboniferous), and crystalline (tonalites and diorites [3]) massif, thus forming one of the intrageosynclinal cordilleras. A submarine landslide, which occured when the coastal cliffs broke off or, what is more likely, after a volcanic eruption, caused these rocks to be inserted into the Lowest Palaeogene sediments of the Sub-Silesian series.

INSTITUTE OF PHYSICAL GEOLOGY, SCHOOL OF MINING AND METALLURGY, CRACOW.

REFERENCES

[1] E. Tietze, Jahrb. geol. Reichsanst., 1891.
[2] M. Książkiewicz, *Objaśnienie ark. Wadowice*, Państw. Inst. Geol. 1951.
[3] T. Wieser, Ann. Soc. Géol. Pol. **22** (1952).

On the Upper Pre-Cambrian (Riphaean) of Poland

by

J. SAMSONOWICZ

Communicated at the meeting of June 20, 1955

This paper deals with deposits directly older than the Lower Cambrian, recorded from two regions, namely the Holy Cross Mountains in Central Poland and the Sudeten (the Kaczawa chain).

The Holy Cross Mountains form a large anticlinorium the base of which is built up of Cambrian deposits consisting of four anticlinal belts running almost equatorially. The longitudinal axis of this massif, about 100 km. in length, is inclined to the West. Most conspicuous is the morphology of the northern belt built up of Middle and Upper Cambrian rocks. Lower and Middle Cambrian rocks occur in the intervening belt, while two southern belts consist of Lower Cambrian rocks only.

As the longitudinal axis of the mountain massif is raised to the East, the three latter belts gradually widen out and unite in that direction, the Lower Cambrian rocks gaining importance as constituents. In the SE sector of the mountain massif, non-fossiliferous rocks, differing in composition and pattern, crop out from beneath the *Holmia* horizon deposits of the Lower Cambrian. Their presence has recently been recorded in a number of deep borings over a large area extending 85 km. in length to the SE from the furthermost easterly Cambrain outcrops of the Holy Cross Mountains, on the River Vistula. Here, in a deep substratum, they form a wide ridge thickly roofed by the Tertiary (Miocene), while its slopes are mantled by Tertiary, Mesozoic and Upper Palaeozoic deposits (Devonian, Lower Carboniferous).

These deposits, belonging to an age undoubtedly older than the Lower Cambrian are chiefly argillaceous, containing sandstone intercalations, occasionally quartzites. In coloration they are mostly ashy-greenish. These rocks disclose a strong schistosity, some are sericitous and resemble phyllites, from which it is deduced that they have undergone epizonal metamorphism. They have a very steep dip and have certainly been affected by stronger tectonic disturbances than the Cambrian deposits in the Holy Cross Mountains.

It seems that there is angular discordance between deposits of the pre-Cambrian and those of the Lower Cambrian, suggesting a sedimentary hiatus. The thickness of the pre-Cambrian is not known but presumably it reaches many hundred metres.

The author is of the opinion that these deposits, held by him as directly older than the Lower Cambrian, might most fittingly be called Riphaean, a name introduced by the Academician N. S. Schatsky [1], [2] to describe deposits of common occurrence in the substratum of the Palaeozoic on the Russian Platform, which have lately been made the subject of comprehensive studies. Schatsky is, in fact, prepared to recognise the Riphaean as constituting a g r o u p, independent of either the Proterozoic or of the Palaeozoic group. Other scientists, e.g. B. S. Sokolov [3] are in favour of these deposits retaining the name of Grabau's "Sinian system" and of including the latter in the Palaeozoic group. This question, not only of formal but of wider significance could be settled on grounds of adequate material, naturally not available in Poland.

It has long been known that green phyllitic rocks are constituents of Zuber's "Dobrogea ridge" [4], [5] reposing in the substratum of the Eastern Carpathians and in that of their Foreland, also called the "metacarpathic ridge" [6]. Hypothetically it links the Holy Cross Mountains with Dobrogea. According to Książkiewicz [7] this ridge was still marked in the Upper Malm by the deposits of which it was transgressively overlain.

Similar phyllitic slates build up Central Dobrogea. Already Murgoci [8] has assigned them to the pre-Cambrian, which he compares with the pre-Cambrian of Anatolia; this has been confirmed by Paeckelmann [9] who suggested that these deposits are of Algonkian age, both in Dobrogea and in Anatolia. Paeckelmann also notes that a powerful series of green slates in the eastern Banat described in 1932 by Codarcea, is referable to a similar age.

The author presumes that green argillites of the metacarpathic ridge extending from the Holy Cross Mountains as far south-east as Dobrogea, are of Riphaean age and that analogous deposits in the eastern Banat and in Anatolia may be correlated with them.

In the Kaczawa chain of the Sudeten Mountains the Algonkian age has been assigned to the so-called Radzimowice beds (Altenberger Schichten) [10], [11] displaying lithological resemblance to the pre-Cambrian of Thuringia and Czechoslovakia. Directly younger are the crystalline limestones of Wojcieszów (Kauffunger Kalke), with a thickness of several hundred metres. The main part of these limestones has rightly been referred to the Lower Cambrian [12], [13] on grounds of analogy with Łużyce (the Upper Lusatian area) where these limestones, as well as red shales linked with them, were found to be fossiliferous; the shales have yielded a *Protolenus* fauna.

The Radzimowice beds are argillaceous graywackes with intercalations of phyllites and limestones of the Wojcieszów type. Though they are strongly disturbed tectonically, and their contact with the Wojcieszów limestone is tectonical, as evidenced by the research studies of H. Teisseyre and K. Smulikowski [14], yet they seem directly older than the Wojcieszów limestones and hence referable to the Riphaean.

Equivalents of the Radzimowice beds also occur in other parts of the Sudeten Mountains in association with Wojcieszów limestones e.g. in the Kłodzko area, among a series of intensely metamorphosed rocks [15], [16], [17].

It is quite reasonable to suppose that the pre-Cambrian rocks of the Sudeten Mountains underlying Wojcieszów limestones, as well as analogous deposits in Thuringia and in the Barrandian of Czechoslovakia, are also referable to the Riphaean.

LABORATORY OF STRATIGRAPHIC GEOLOGY, WARSAW UNIVERSITY.

REFERENCES

[1] N. S. Schatsky, *O drevnieyshih otlojenyah osadochnogo chehla russkoi platformy i ob yeye strukturie w drevnieem paleozoie*, Isv. Ak. Nauk USSR, Geolog. Series (1952), No. 1.

[2] — *O granitsie miejdu paleozoiem i protierozoiem i o rifieyskih otlojeniach russkoi platformy*, ibid. (1952), No. 5.

[3] B. S. Sokolov, *O vozrastie drevnieyshego osadochnogo pokrova russkoi platformy*, ibid. (1952), No. 5.

[4] R. Zuber, *Neue Karpathenstudien*, Jb. geol. R. A., Wien 1902.

[5] R. Zuber, *Flisz i nafta*, Lwów 1918.

[6] J. Nowak, *Esquisse de la tectonique de la Pologne*, Cracovie 1927.

[7] M. Książkiewicz, *The Jurassic and the Cretaceous of Bachowice*, Ann. Soc. Géol. Pol. (1955) (in press).

[8] G. Murgoci, *Recherches géol. dans la Dobrogea du Nord*. An. Inst. geol. al. Rom. **5**, (1914).

[9] W. Paeckelmann, *Probleme des Varistikums der Dobrudscha*, Z.d.D.G.G. **87** (1935).

[10] E. Bederke, *Probleme der Sudetengeologie*, Jahresb. d. Schl. Ges. f. vaterl. Cultur **105**. Breslau 1933.

[11] M. Schwarzbach, *Das Bober-Katzbach-Gebirge in Rahmen des europäischen Paläozoikums*, Z.d.D.G.G. **92**, (1940).

[12] M. Schwarzbach, *Das Cambrium der Oberlausitz*, Abh. d. Naturf. Ges. zu Görlitz **32** (1934).

[13] M. Schwarzbach, *Oberlausitzer Schiefergebirge und Boberkatzbachgebirge*, Ibidem, **32** (1936).

[14] H. Teisseyre and K. Smulikowski, *Przekrój przez pd. część Gór Kaczawskich w okolicy Bolkowa*, Przewodnik XXIV Zjazdu P. Twa Geol. w Sudetach w r. 1951.

[15] G. Fischer, *Das Dach des Moldanubikums im Schlesien*, Jb. pr. geol. L.A. **56** Berlin 1936.

[16] E. F. Vangerow *Das Normalprofil des Alkgonkiums und Kambriums in dew mittleren Sudeten*, Geol. Rundschau **34**, (1943).

[17] K. Smulikowski, *The old crystalline formations of the Sudeten Mountains*, Ann. Soc. Géol. Pol. **21** (1951).